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$$\frac{s\sqrt{[(rv+s)^2+2v(rv+s)+1]}}{rv+v+s}=\frac{2s}{rv+s}.$$

Whence $\sqrt{[(rv+s)^4+2v(rv+s)^3+(rv+s)^2]}=2(rv+v+s)$.

Because v cannot be eliminated from this equation therefore the trisection cannot, in general, be exact; otherwise we would have an equation of the 4th degree with an infinite number of roots.

Moreover, as the above equation is verified by substituting for v its value at either limit, and as it has two negative roots, it follows that the foregoing constrction will not trisect *any* finite angle except one of 90°.—Ed.]

SOLUTION OF A PROBLEM IN PROBABILITIES.

BY THE EDITOR.

If four bricks are placed on each other at random, with their longest axes horizontal and in the same vertical plane, determine the probability that the pile will stand.*

Let the bricks, in the order in which they are placed on the pile, be represented respectively by (1), (2), (3), (4).

Denote the length of a brick by a, and the horizontal distance between the centers of (1) and (2), (2) and (3), (3) and (4), respectively, by x, y, z.

Then, if x be not greater than $\frac{1}{6}a$, and if y is estimated in the same direction from the center of (1) as x is, and is taken not greater than $\frac{1}{4}a$, the center of (4) may take any position on (3) and the pile will stand. This 1st case therefore gives, for the number of stable positions of the pile,

$$\tfrac{1}{6}a\times\tfrac{1}{4}a\times a=u_1=\tfrac{1}{24}a^3.$$

Also, while x is restricted to $\frac{1}{6}a$, y may take any value between $\frac{1}{4}a$ and $\frac{1}{2}a$, but the *range* of the center of (4), for stable positions of the pile, will now be $\frac{1}{2}(3a-4y)$; hence the number of stable positions in this case will be

$$\tfrac{1}{6}a\int_{\frac{1}{4}a}^{\frac{1}{2}a}\tfrac{1}{2}(3a-4y)dy=\tfrac{1}{6}a\Big[\tfrac{3}{2}ay-y^2\Big]_{\frac{1}{4}a}^{\frac{1}{2}a}=u_2=\tfrac{1}{32}a^3.$$

In like manner we may find that for values of x between $\frac{1}{6}a$ and $\frac{1}{4}a$, y may have any value between 0 and $\frac{1}{2}(a-3x)$, and z any value from $-\frac{1}{2}a$ to $+\frac{1}{2}a$; hence the number of stable positions will be

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*This problem was, I believe, first proposed by Artemas Martin, in the Schoolday Visitor for Dec. 1871. No solution was published in that periodical, nor elsewhere so far as I know, though two other solutions of the question have been made, independently; one by E. B. Seitz and one by Henry Heaton.

$$a\int_{\frac{1}{6}a}^{\frac{1}{4}a}\int_{0}^{\frac{1}{2}(a-3x)} dx\,dy=\tfrac{1}{2}a\Big[\,ax-\tfrac{3}{2}x^2\,\Big]_{\frac{1}{6}a}^{\frac{1}{4}a}=u_3=\tfrac{1}{64}a^3.$$

Also, for the same values of x, if y be taken between the limits $\frac{1}{2}(a-3x)$ and $\frac{1}{2}a$, the pile will stand for all positions of the center of (4) included in the expression $2a-3x-2y$. Therefore, in this case we have

$$\int_{\frac{1}{6}a}^{\frac{1}{4}a}\int_{\frac{1}{2}(a-3x)}^{\frac{1}{2}a}(2a-3x-2y)dx\,dy=\Big[\,\tfrac{3}{4}ax^2-\tfrac{3}{4}x^3\,\Big]_{\frac{1}{6}a}^{\frac{1}{4}a}=u_4=\tfrac{41}{2304}a^3.$$

If for values of x between $\frac{1}{4}a$ and $\frac{1}{3}a$ y be taken between the limits 0 and $\frac{1}{2}(a-3x)$ the range of the center of (4) for stable positions of the pile will again be a, and we shall have

$$a\int_{\frac{1}{4}a}^{\frac{1}{3}a}\int_{0}^{\frac{1}{2}(a-3x)} dx\,dy=\tfrac{1}{2}a\Big[\,ax-\tfrac{3}{4}x^2\,\Big]_{\frac{1}{2}a}^{\frac{1}{3}a}=u_5=\tfrac{1}{192}a^3.$$

Also if for the same values of x as in the last case y take any value between $\frac{1}{2}(a-3x)$ and $a-2x$, in all case of stability of the pile, the center of (4) will again be confined to the line $2a-3x-2y$, and we shall have

$$\int_{\frac{1}{4}a}^{\frac{1}{3}a}\int_{\frac{1}{2}(a-3x)}^{a-2x}(2a-3x-2y)dx\,dy=\Big[\,\tfrac{1}{4}a^2x-\tfrac{1}{12}x^3\,\Big]_{\frac{1}{4}a}^{\frac{1}{3}a}=u_6=\tfrac{395}{20736}a^3.$$

If we allow x to take any value between $\frac{1}{3}a$ and $\frac{1}{2}a$, y may have any value between 0 and $2a-x$, and the pile will stand for all postions of the center of (4) as in the last case, and we shall have

$$\int_{\frac{1}{3}a}^{\frac{1}{2}a}\int_{0}^{a-2x}(2a-3x-2y)dx\,dy=\Big[\,a^2x-\tfrac{3}{2}ax^2+\tfrac{2}{3}x^3\,\Big]_{\frac{1}{3}a}^{\frac{1}{2}a}=u_7=\tfrac{352}{20736}.$$

If while x is estimated in either direction from the center of (1) y is taken in the *opposite* direction, the stable positions of the pile will be as follows:—If x be taken between 0 and $\frac{1}{3}a$, and y, between 0 and $\frac{1}{4}a$, the center of (4) may take any position on (3) Hence we have

$$\tfrac{1}{3}a\times\tfrac{1}{4}a\times a=u_8=\tfrac{1}{12}a^3.$$

If x be taken between 0 and $\frac{1}{2}a$, and y, between $\frac{1}{4}a$ and $\frac{1}{2}a$ the center of (4) must be confined to the line $\frac{1}{2}(3a-2y)$, and we get

$$\tfrac{1}{2}a\int_{\frac{1}{4}a}^{\frac{1}{2}a}\tfrac{1}{2}(3a-4y)dy=\tfrac{1}{2}a\Big[\,\tfrac{3}{2}ay-y^2\,\Big]_{\frac{1}{4}a}^{\frac{1}{2}a}=u_9=\tfrac{3}{32}a^3.$$

There remains only the case in which x varies from $\frac{1}{3}a$ to $\frac{1}{2}a$, and y, from 0 to $\frac{1}{4}a$; and in this case, if, while x takes any value from $\frac{1}{3}a$ to $\frac{1}{2}a$, y vary within the limits 0 and $\frac{3}{2}x-\frac{1}{2}a$, the center of (4) will be confined to the line $2a-3x+2y$, and we shall have

$$\int_{\frac{1}{3}a}^{\frac{1}{2}a}\int_{0}^{\frac{1}{2}(3x-a)}(2a-3x+2y)dxdy=\Big[\,-\tfrac{3}{4}x^3+\tfrac{3}{2}ax^2-a^2x\,\Big]_{\frac{1}{3}a}^{\frac{1}{2}a}=u_{10}-\tfrac{5}{288}a^3.$$

Finally, with the same limits for x as in the last case, if y vary from $\frac{3}{2}x-\frac{1}{2}a$ to $\frac{1}{4}a$ the center of (4) may take any position on (3), and we shall havs

$$a\int_{\frac{1}{3}a}^{\frac{1}{2}a}\int_{\frac{1}{2}(3x-a)}^{\frac{1}{4}a} dxdy = \tfrac{3}{4}ax^2 = u_{11} = \tfrac{1}{48}a^3.$$

As the whole number of different trials that can be made will be represented by $8a^3$, if P represent the required probability, because the stable positions above calculated are just half the whole number of stable positions, (x having been taken only from the center of (1) to *one* extremity,) we shall have

$$P = \frac{u_1 + u_2 + \ldots\ldots + u_{11}}{4a^3} = \frac{209}{2304}.$$

SOLUTION OF TWO PROBLEMS IN SUMMATION OF SERIES.

BY PROF. D. TROWBRIDGE, WATERBURGH, N. Y.

1. REQUIRED the sum of the series

$$1 + \frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r} + \ldots + \frac{1}{1+(n-1)r} = \Sigma_n^{(r)} \ldots\ldots (1)$$

Solution.—First find the value of the definite integral

$$\frac{1}{r}\int_{-1}^{0} y^p dy (1+y)^{\frac{1}{r}-1} = \Big[_{-1}^{0} y^p(1+y)^{\frac{1}{r}} - \frac{py^{p-1}r(1+y)^{(1\div r)-1}}{1+r}$$

$$+ \frac{p(p-1)y^{p-2}r^2(1+y)^{(1\div r)+2}}{(1+r)(1+2r)} + (-1)^p . \frac{p(p-1)\ldots 2.1.r^p(1+y)^{(1\div r)+p}}{(1+r)(1+2r)\ldots(1+pr)}\Big]$$

$$= (-1)^p . \frac{p(p-1)\ldots 2.1.r^p}{(1+r)(1+2r)\ldots(1+pr)} = V_p \ldots\ldots (2)$$

Make $p = 0, 1, 2, 3,$ &c., successively, and we shall have

$$V_0 = 1,\ V_1 = -\frac{r}{1+r},\ V_2 = \frac{2r^2}{(1+r)(1+2r)},$$

$$V_3 = -\frac{3.2r^3}{(1+r)(1+2r)(1+3r)}, \text{ \&c.} \ldots\ldots (3)$$

Now take the series

$$1 + x^r + x^{2r} + \ldots\ldots + x^{(n-1)r} = \frac{x^{nr}-1}{x^r-1}, \qquad \text{then}$$

$$\Sigma_n^{(r)} = \int_0^1 dx(1 + x^r + x^{2r} + \ldots) = \int_0^1 dx. \frac{x^{nr}-1}{x^r-1}.$$

Put $x^r = 1 + y$, $x = \sqrt[r]{(1+y)}$, $dx = (1\div r)\times \sqrt[r-1]{(1+y)}dy$, and

$$\frac{x^{nr}-1}{x^r-1} = \frac{(1+y)^n-1}{y} = n + \frac{n(n-1)y}{1\,.\,2} + \frac{n(n-1)(n-2)y^2}{1\,.\,2\,.\,3} + \ldots$$